Triangulation: Why Optimize?

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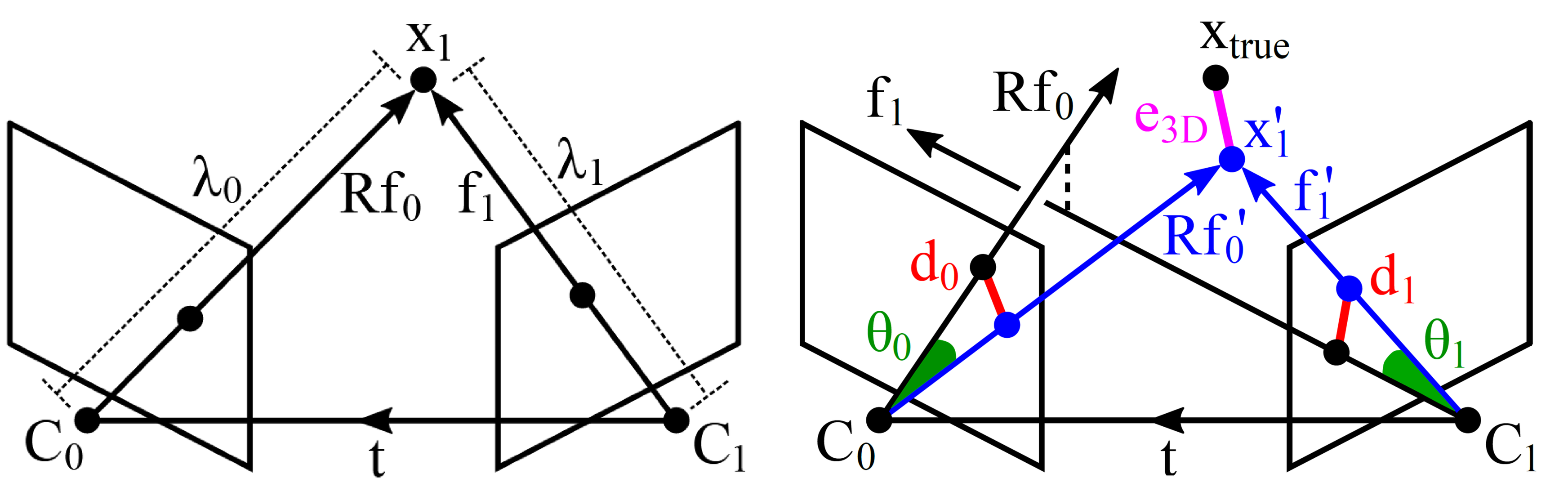
1. Two-View Triangulation: Locating the 3D point given its projections in two views with known calibration and pose.

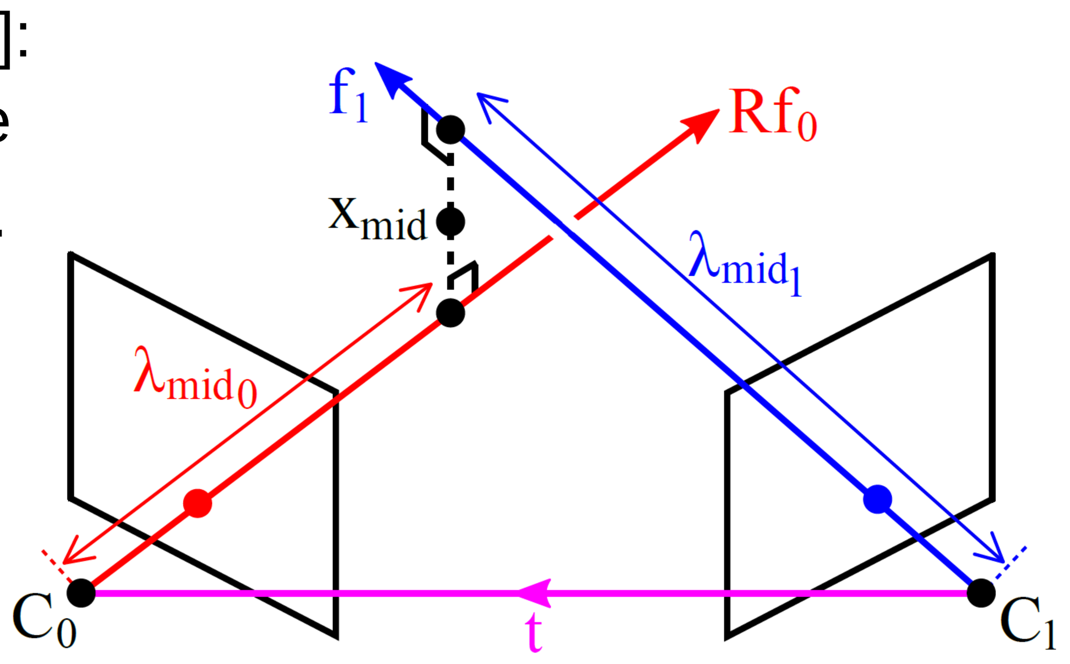


2. Optimal Method: Correct the rays (f_0 and f_1) to make them intersect with a minimal image/angular reprojection cost, e.g.,

- L_1 norm: $d_0 + d_1$ [1] or $\theta_0 + \theta_1$ [4]
- L_2 norm: $d_0^2 + d_1^2$ [1,3] or $\sin^2(\theta_0) + \sin^2(\theta_1)$ [4]
- L_∞ norm: $\max(d_0 + d_1)$ [2] or $\max(\theta_0 + \theta_1)$ [4]

3. Midpoint Method [1]:
Find the midpoint of the common perpendicular.

$$\lambda_{mid0} = \frac{\hat{\mathbf{p}} \cdot \mathbf{r}}{\|\mathbf{p}\|}, \quad \lambda_{mid1} = \frac{\hat{\mathbf{p}} \cdot \mathbf{q}}{\|\mathbf{p}\|} \quad (1)$$



where $\mathbf{p} := \mathbf{R}\hat{\mathbf{f}}_0 \times \hat{\mathbf{f}}_1$, $\mathbf{q} := \mathbf{R}\hat{\mathbf{f}}_0 \times \mathbf{t}$, $\mathbf{r} := \hat{\mathbf{f}}_1 \times \mathbf{t}$.

4. Proposed Alternative Midpoint Method:

$$\lambda_0 = \frac{\|\mathbf{r}\|}{\|\mathbf{p}\|}, \quad \lambda_1 = \frac{\|\mathbf{q}\|}{\|\mathbf{p}\|} \quad (2)$$

- When the rays intersect, this is basically the sine-rule.
- $\lambda_{mid0} \leq \lambda_0$ and $\lambda_{mid1} \leq \lambda_1$.

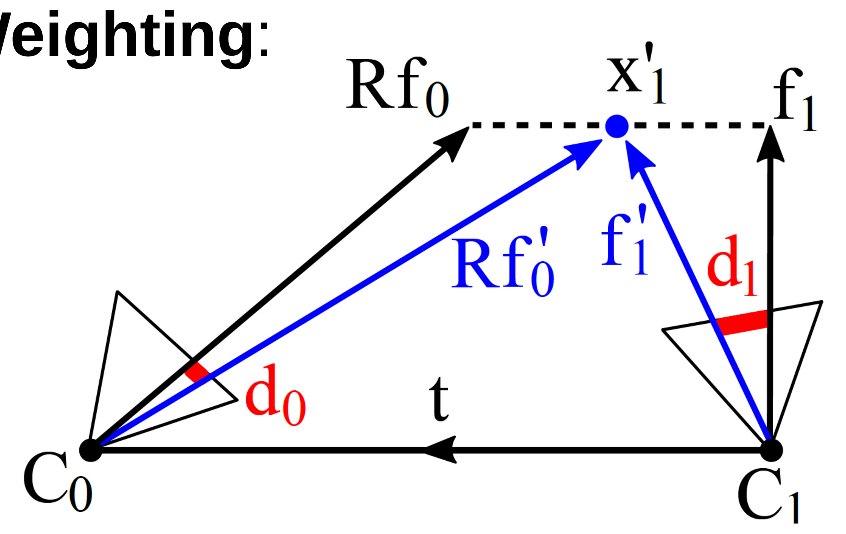
5. Proposed Alternative Cheirality Check:

- In contrast to Eq (1), Eq (2) is unsigned.
  → Need an alternative cheirality check!
- We discard the point if assuming a negative depth brings the two points ($\mathbf{t} + \lambda_0 \mathbf{R}\hat{\mathbf{f}}_0$ and $\lambda_1 \hat{\mathbf{f}}_1$) closer to each other.

6. Proposed Inverse Depth Weighting:

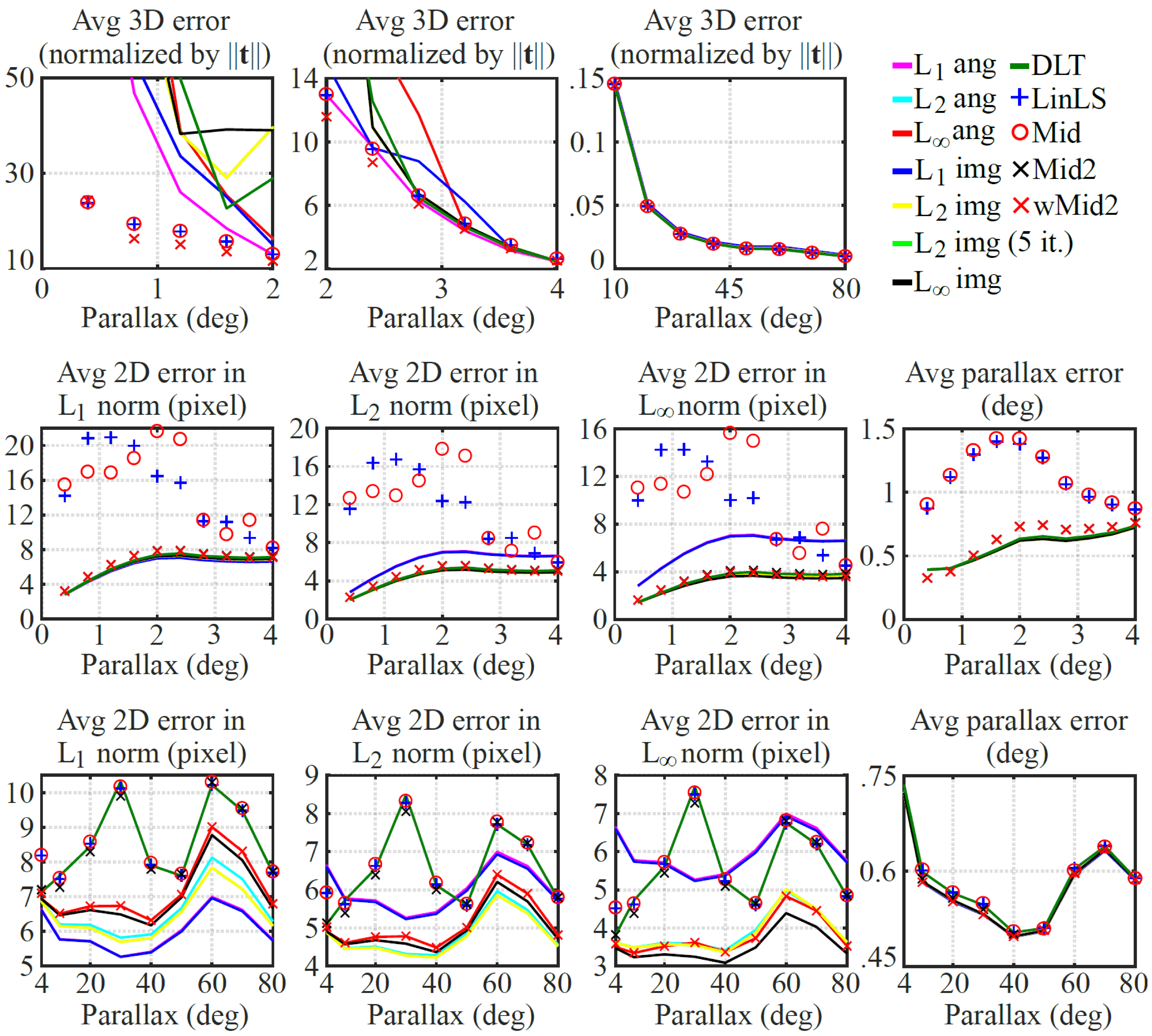
- Unweighted:

$$x'_1 = \frac{\mathbf{t} + \lambda_0 \mathbf{R}\hat{\mathbf{f}}_0 + \lambda_1 \hat{\mathbf{f}}_1}{2}$$



- Inverse Depth Weighted:

$$x'_1 = \frac{\lambda_0^{-1}(\mathbf{t} + \lambda_0 \mathbf{R}\hat{\mathbf{f}}_0) + \lambda_1^{-1}(\lambda_1 \hat{\mathbf{f}}_1)}{\lambda_0^{-1} + \lambda_1^{-1}}$$



7. Evaluation on synthetic data:

As reported earlier in [1], we found that:

1. Lower parallax leads to larger 3D errors.
2. All methods show similar 3D accuracy for large parallax angles (> 4 deg).
3. 2D and 3D errors are not well correlated.

Additionally, we found that:

1. The classic midpoint method is biased to overestimate the small parallax angles (< 4 deg). Our methods are less biased.
2. Our weighted midpoint method achieves the best overall accuracy in 3D and parallax estimation. Also, it shows similar 2D accuracy to that of L_∞ methods.
3. Our methods achieve comprable speed to that of the state-of-the-art.

Method	Points/sec
Classic Midpoint (Mid) [1]	38M
L_1 angular [4]	29M
Our Midpoint (Mid2)	**21M**
L_∞ angular [4]	13M
Our Weighted Midpoint (wMid2)	**12M**
L_2 image with 5 iterations [3]	550K

[1] R. Hartley and P. Sturm. *Triangulation*. Comput. Vis. Image. Underst. 1997

[2] D. Níster. *Automatic Dense Reconstruction from Uncalibrated Video Sequences*. PhD thesis, 2001

[3] P. Lindstrom. Triangulation made easy, CVPR 2010

[4] S. Lee and J. Civera. Closed-Form optimal two-view triangulation based on angular errors. ICCV 2019